scribed in Step 3 in the following suggested 3-step procedure for obtaining throat pressure ratio.

Steps 1 and 2 are to be used for all throat calculations while Step 3 is to be used only for the special case when discontinuities are present at the throat.

A suggested procedure

Step 1: Obtain an initial estimate for throat pressure ratio from the approximate formula

$$P_c/P_t = [(\gamma_S + 1)/2]^{\gamma_S/(\gamma_S - 1)} \quad (15)$$

using the value of γ_S corresponding to the combustion conditions. For the example in Table 1, using $\gamma_S = 1.2149$, Eq. (15) gives as a first estimate $P_c/P_t = 1.7806$. Equilibrium calculations for this estimated pressure ratio are shown in the second column of Table 1. The throat temperature for this first estimate is 2315°K (the melting point of Al_2O_3) with mole fractions of Al_2O_3(l) = 0.03320 and Al_2O_3(s) = 0.00900.

Step 2: Obtain a second estimate for throat pressure ratio. If the method of Ref. 1 is used, the next estimate is P_c/P_t = 1.6151. At this pressure ratio, T_t is 2332°K which is above the melting point of 2315°K and no Al_2O_3(s) is present. The sequence of Steps 1 and 2 (namely, liquid and solid at one estimate of throat pressure ratio and liquid only at the next estimate) indicates that discontinuities exist at the throat.

Step 3: Estimate the pressure ratio at the melting point where the solid phase just begins to appear. This may be done by using the following expression:

$$\ln P_t = \ln P + (\partial \ln P/\partial \ln T)_S(\ln T_{m.p.} - \ln T) \quad (16)$$

where[3]

$$(\partial \ln P/\partial \ln T)_S = c_P/nR(\partial \ln v/\partial \ln T)_P \quad (17)$$

In Eqs. (16) and (17), the values for the right hand side are for the previous point, $P_c/P_t = 1.6151$ [no Al_2O_3(s)]. Equation (17) gives a value $(\partial \ln P/\partial \ln T)_S = 5.5059$. This value in Eq. (16) gives a final throat estimate of $P_t = 4.1003 \times 10^6$ N/m² or $P_c/P_t = 1.6815$. At this pressure ratio, equilibrium calculations gave T = 2315°K and a mole fraction of 0.00001 for Al_2O_3(s), as may be seen in Table 1 in the column labeled "Throat."

6. Discussion

For the assumed model, the correct throat pressure ratio giving zero mole fraction for Al_2O_3(s) at T = 2315°K is 1.6814. Therefore, for this example, the suggested method located the throat pressure ratio to within 0.0001 (0.006%) of the correct value which is in excellent agreement. For several other cases that were tried and that involved a much longer extrapolation in Eq. (16) than in this case, the throat pressure ratio was within 0.0005 (0.03%) of the correct pressure ratio which is still excellent.

The calculation of throat conditions for the special circumstances discussed in this paper has been incorporated as part of a general computer program for chemical equilibrium calculations. The program, which includes several applications, is available from the author.

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Structural Optimization of a Panel Flutter Problem

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Nomenclature

a_i	= modal amplitudes
b	= segment length
c	= panel length
$D(X)$	= panel stiffness
D_o	= reference stiffness of uniform panel
g_o	= aerodynamic damping parameter = $\rho Uc^2(M^2 - 2)/[(M^2-1)^{3/2}(D_o m_o)^{1/2}]$
M	= Mach number
$m(X),m_i$	= panel mass per unit length
$\bar{m}(X),\bar{m}_i$	= optimal values of m,m_i
m_o	= reference mass of uniform panel
P	= midplane compressive load
p	= dimensionless load = Pc^2/D_o
p_o	= critical value of p for flutter
$\bar{p},\bar{p}_o$	= approximate critical values of p
T	= time
t	= dimensionless time = $T(D_o/m_o)^{1/2}/c^2$
U	= speed of supersonic flow
$\bar{W}$	= total weight of optimum panel
W_o	= total weight of reference uniform panel
$W(X,T)$	= panel deflection
$w(x,t)$	= dimensionless panel deflection = W/c
$w_i(x)$	= deflection modes
X	= coordinate along length
x	= dimensionless coordinate = X/c
α^2,β^2	= parameters relating mass and stiffness
γ	= $[(1-\alpha^2)/\beta^2]^{1/2}$
$\delta(x),\delta_i$	= dimensionless panel stiffness; $\delta = D/D_o$
$\mu(x),\mu_i$	= dimensionless mass; $\mu = m/m_o$
$\bar{\mu}(x),\bar{\mu}_i$	= optimal values of μ,μ_i
θ	= frequency parameter
λ_o	= dynamic pressure parameter = $\rho U^2c^3/[D_o(M^2-1)^{1/2}]$; $\lambda_o > 125$
ρ	= density of air
$(\ )'$	= $d(\ )/dx$

Introduction

Recent reviews by Ashley[1] and Dowell[2] have described the present status of research on panel flutter. A new development concerns the structural optimization of panels in order to minimize the panel weight while satisfying certain flutter requirements. This problem has been considered by Turner[3] and by McIntosh, Weisshaar, and Ashley.[4] An approximate method for obtaining the optimal panel design is proposed here, and calculations are carried out for a panel with segment-wise constant mass distribution. The results, although only approximate, indicate that significant savings in weight may be possible with the use of nonuniform panels.

Analysis

Consider a flat elastic panel of infinite span, as shown in Fig. 1, subjected to a supersonic flow over one side and a uniform midplane compressive load P. The equation of motion based on first-order theory is given by[5]

$$\frac{\partial^2}{\partial X^2}\left[D(X)\frac{\partial^2 W}{\partial X^2}\right] + m(X)\frac{\partial^2 W}{\partial T^2} + P\frac{\partial^2 W}{\partial X^2} + \frac{\rho U^2}{(M^2-1)^{1/2}}\left[\frac{\partial W}{\partial X} + \frac{(M^2-2)}{U(M^2-1)}\frac{\partial W}{\partial T}\right] = 0 \quad (1)$$

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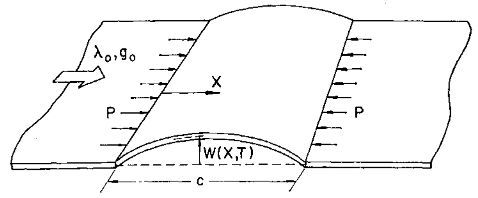

Fig. 1 Geometry of two-dimensional panel.

In dimensionless form this equation becomes

$$\frac{\partial^2}{\partial x^2}\left[\delta(x)\frac{\partial^2 w}{\partial x^2}\right] + \mu(x)\frac{\partial^2 w}{\partial t^2} + p\frac{\partial^2 w}{\partial x^2} + \lambda_0 \frac{\partial w}{\partial x} + g_0 \frac{\partial w}{\partial t} = 0 \quad (2)$$

with boundary conditions

$$w = \delta(x)\partial^2 w/\partial x^2 = 0 \quad \text{at} \quad x = 0,1 \quad (3)$$

The stiffness parameter δ is assumed to be a given function of the mass parameter μ.

For a uniform panel [i.e., $\mu(x) \equiv 1$], and for given values of the dynamic pressure parameter λ_o and the aerodynamic damping parameter g_o, there is a critical value p_o of the load parameter p at which flutter occurs. The problem at hand is to determine the design $\mu(x)$ that leads to the same flutter

$$\int_0^1 \mu(x)dx$$

conditions λ_o, g_o, p_o, and for which the quantity is minimum. In other words, which design satisfies given flutter requirements and has minimum total weight?

A set of differential equations involving the optimal design of $\mu(x)$ may be obtained with the use of the calculus of variations and Lagrange multipliers,[3,4] and approximate techniques may then be applied to solve these equations. A different procedure is proposed here. For given λ_o and g_o, an approximate critical value $\bar{p}$ of the load parameter p is derived with the use of Galerkin's method. This load $\bar{p}$ is a function of the design $\mu(x)$, and its value for the uniform panel is denoted by $\bar{p}_o$. The designs $\mu(x)$ are then varied, and the one which leads to the same critical value $\bar{p}_o$ as the uniform panel and has minimum total weight is chosen as the (approximate) optimal design $\bar{\mu}(x)$. In other words, this procedure makes use of an approximate critical load for which an analytical expression is known, rather than the exact critical load.

In order to obtain a simple expression for $\bar{p}$, the deflection $w(x,t)$ is assumed to have the form

$$w(x,t) = [a_1 w_1(x) + a_2 w_2(x)]e^{\theta t} \quad (4)$$

where w_1 and w_2 satisfy the boundary conditions of Eq. (3). Galerkin's method furnishes the following two linear equations in a_1 and a_2[6]:

$$\sum_{j=1}^{2} a_j \int_0^1 w_i[(\delta w_j'')'' + p w_j'' + \lambda_0 w_j' + (\mu\theta^2 + g_0\theta)w_j]dx = 0, \quad i = 1,2 \quad (5)$$

The condition for Eqs. (5) to possess a nontrivial solution is that the determinant of the coefficients of a_1 and a_2 be zero, which yields a fourth-order polynomial in θ. The Routh-Hurwitz conditions are employed next to obtain the critical value $\bar{p}$, below which the four roots θ have negative real parts and the panel is asymptotically stable. The governing condition is simply a quadratic equation in $\bar{p}$. Once the expression for $\bar{p}$ is determined, the optimal design $\bar{\mu}(x)$ may be calculated.

Example

As an example, consider a sandwich panel whose mass is a linear function of its stiffness and is distributed symmetrically about $x = \frac{1}{2}$ with

$$\mu(x) \equiv \mu_1 = \alpha^2 + \beta^2\delta_1 \quad \text{for} \quad 0 < x < (b/c)$$

$$\mu(x) \equiv \mu_2 = \alpha^2 + \beta^2\delta_2 \quad \text{for} \quad (b/c) < x < 1 - (b/c)$$

$$\mu(x) \equiv \mu_1 = \alpha^2 + \beta^2\delta_1 \quad \text{for} \quad 1 - (b/c) < x < 1$$

The functions $w_1(x)$ and $w_2(x)$ are chosen as

$$w_1(x) = \sin\pi x, \quad w_2(x) = \sin 2\pi x$$

and the quantities λ_o and g_o as $\lambda_o = 400\,\gamma^2$, $g_0 = \pi^2\gamma$ where $\gamma = [(1 - \alpha^2)/\beta^2]^{1/2}$. In order to avoid the unrealistic case of zero stiffness, the constraint $\delta_i \geq 0.2/\beta^2$, $i = 1,2$, is included.

Galerkin's method as previously described yields an expression for the critical value $\bar{p}(\mu_1,\mu_2)$. In the case of uniform mass, $\mu_1 = \mu_2 = 1$ and $\bar{p}_0 = -1.9\pi^2$. The values $\bar{\mu}_1$ and $\bar{\mu}_2$, which satisfy $\bar{p}(\bar{\mu}_1,\bar{\mu}_2) = \bar{p}_o$ and minimize

$$\int_0^1 \mu(x)dx$$

can be easily determined numerically, keeping in mind the constraints of constant total weight and a lower bound on the stiffness. The resulting ratio $\bar{\mu}_2/\bar{\mu}_1$ ($= \bar{m}_2/\bar{m}_1$) is depicted in Fig. 2 as a function of b/c for various values of α^2. The ratio $\bar{W}/W_o$ of the total weight $\bar{W}$ of this optimum panel to the total weight W_o of the uniform panel with the same flutter conditions is shown in Fig. 3.

The results of this particular example indicate that a considerable saving in weight may be possible. If $\alpha^2 = 0.2$, for instance, then the minimum-weight panel satisfying the given flutter conditions is obtained by choosing $b = 0.22c$ and $m_2 = 3.0m_1$, and its weight is about 16% less than that

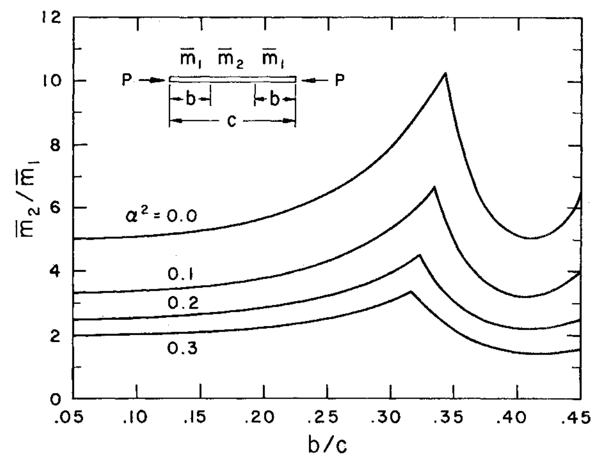

Fig. 2 Mass ratio for optimal design.

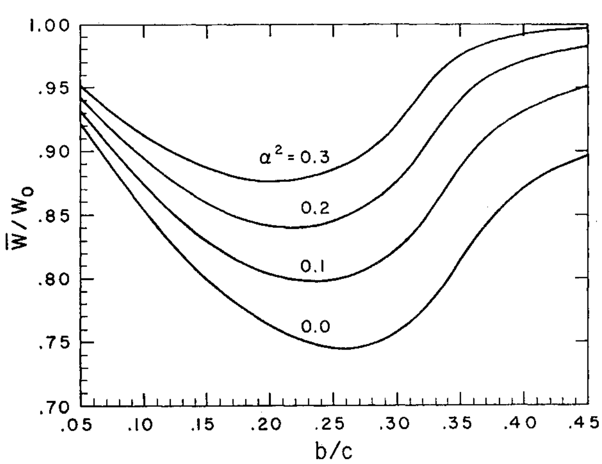

Fig. 3 Ratio of optimal design weight to uniform panel weight.

of the corresponding uniform panel. The particular values shown in Figs. 2 and 3, however, may not be extremely accurate due to the nature of the approximate method and the two-mode deflection assumption. It is hoped, however, that these results may provide a starting point for further analysis and design by other methods.

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Dark-Pause Measurements in a High-Pressure Arc Discharge

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THE experiments on exploding wires are fairly complete,[1] but the data on dwell and restrike characteristics of a confined arc in a closed chamber are fragmentary, particularly at pressures above 1 atm. In this paper, we examine the dwell or dark-pause data compiled through 2 years of tests of an arc-heated, shock tube driver at the Ames Research Center.[2,3] In this driver, helium (or other) gas under several atmospheres of pressure in the arc chamber (9.5 cm diam) is heated at constant volume by an electrical arc discharge. Energy for the arc is supplied from a one megajoule, 40 kv capacitor bank. On the basis of our data, it is possible to give at least a qualitative interpretation of the dwell period. The data cover a wide range of pressures and gases for three different arc lengths—76, 137, and 290 cm.

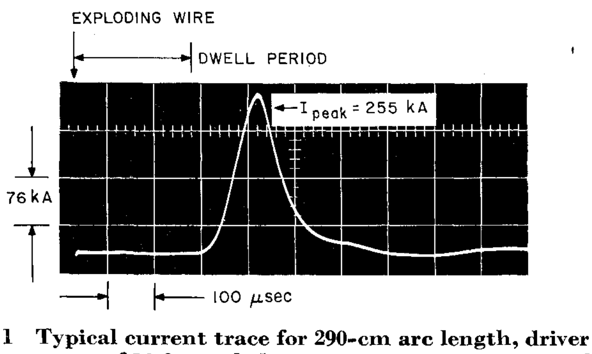

Fig. 1 Typical current trace for 290-cm arc length, driver load pressure of 10.2 atm helium, tungsten trigger wire of 0.127-mm diam, preset voltage of 40 kv.

To initiate an arc discharge between widely spaced electrodes at high gas pressures requires the use of an ignition wire bridging the electrode gap. Generally, the wire is stretched or drawn between the electrodes. The general picture of how the trigger wire leads to the eventual arc discharge is described as follows: the initial current surge instantly explodes the wire from its metallic state by ohmic heating to a nonconducting column of hot aerosols at high pressure. The abrupt decrease in the electrical conductivity stops the capacitor discharge with no apparent change in the bank voltage. As a result of the increase in pressure, the column expands and acts on the surrounding gas like a piston, sending into it a cylindrical shock wave. With the gasdynamic expansion, the pressure (and density) of the vapor decreases in the central region of symmetry. When the pressure reaches a critical value,[4] avalanche breakdown starts, and the arc strike occurs. The time required for the central core region to reach this critical value of pressure is defined as the dwell period. If the critical value is not reached, the arc will not occur and the system will remain in an open-circuit condition.

The relationship between the breakdown voltage and the product of (pressure) x (electrode spacing) is generally known as Paschen's law. This relationship is discussed in textbooks,[5] together with information that can be used to calculate the minimum breakdown voltage for a particular electrode arrangement, for a given gas and pressure. Under the range of experimental conditions employed, Paschen's law is used to interpret the data together with concepts from cylindrical blast wave theory.

A typical current waveform for an arc length of 290 cm is shown in Fig. 1. From this oscillogram, and others like it, the dwell period was measured as the interval from the wire explosion to the start of the rise of the arc current. The dwell period is terminated once the restrike current flow starts and, thus, the dwell characteristics can be evaluated without regard to subsequent shock tube events; i.e., diaphragm opening, shock-wave development, etc. While the trigger wire was exploded in times apparently less than 1 microsecond, the vaporized metal column does not instantaneously induce the arc strike. With a 76-cm arc length, the duration of the period from explosion to the start of the arc discharge varied from 900 μsec for an 8 kv preset voltage to about 20 μsec at 40-kv preset. Long dwell periods were noted also for the 137- and 290-cm arc lengths. After collecting the dwell data compiled through something like 500 firings, it was found that, if the data were presented under one driver pressure condition, with the same trigger wire arrangement, the dwell period is a function only of the initial field intensity (i.e., preset voltage/arc length). As summarized in Fig. 2, the results for three different arc lengths fall on a single line on the log-log plot. The load pressures of the gases in the three driver chambers were kept at 18.5 atm and the same type of straight trigger wire was used in each—a 0.127-mm-diam tungsten wire. The gases and gas compositions tested were helium, argon, nitrogen, a mixture of 1-atm air plus helium, and mixture ratios of 10–50% argon in helium. Dwell periods for helium (and He/Ar mixtures) were measured from repeated test in the facility, and the run-to-run variation was small (±4%).

Voltage and current records were obtained also over a range of driver pressure operation for two of the driver lengths with the same trigger wire arrangement as noted in connection with Fig. 2. As summarized in Fig. 3, these dwell data show an increasing duration with increasing gas pressure. A

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